What is the EMAS?

The Early Mathematics Assessment System (EMAS) is a reliable and valid research-based assessment of early mathematical thinking that draws on modern cognitive science as well as developmental and educational research. Created by Dr. Herb Ginsburg and colleagues at Teachers College, Columbia University[1], and expanded and adapted by researchers at CASTL, the EMAS is designed to measure a broad range of mathematical content in pre-kindergarten and kindergarten.

Function of the EMAS

There are four main purposes of the EMAS. First, it enables teachers and school/program leadership to measure and describe the mathematical skills of their students at the beginning, middle, and end of the school year. Second, it measures how much children grow in their mathematics skills within an academic year and across pre-kindergarten through kindergarten. Third, it identifies strengths and areas of growth for individual students to help teachers plan instruction. Finally, in aggregate, EMAS data allows schools, programs, and divisions to assess needs related to early mathematics instruction.

EMAS at a glance

- Teachers administer the assessment to students individually, using a flipbook and manipulatives.
- The assessment takes approximately 20–25 minutes per student to administer.
- Items are designed to capture a wide range and variety of early mathematics skills.
- It uses hands-on materials to engage students and to help teachers observe students' thinking.
- The skills and knowledge assessed with the EMAS align, but do not perfectly correspond with Virginia's Early Learning and Development Standards (ELDS; 2021), Virginia Standards of Learning (SOL; 2023), and Clements and Sarama's Mathematics Learning Trajectories (2009)[2].
- The EMAS is not an SOL assessment.
- It is given in the fall and spring of pre-kindergarten and kindergarten, with the option of giving the EMAS at the mid-year time point.

What skills are assessed with the EMAS?

The EMAS focuses on key foundational mathematics skills that set students on a successful early mathematics trajectory. The EMAS is comprised of four subdomains: Geometry, Patterning, Numeracy, and Computation. The number of items in each subdomain varies based on the time of year the assessment is given and differs in pre-kindergarten and kindergarten

[1] Ginsburg, H. P., Pappas, S., & Lee, Y. (2010). Early Mathematics Assessment System. An unpublished assessment measure created as part of the NIH supported project Computer Guided Comprehensive Mathematics Assessment for Young Children (Project number 1 RO1 HD051538-01).

[2] Clements, D. H., & Sarama, J. (2009). Learning and teaching early math: The learning trajectories approach. New York: Routledge.

Module 1	Module 2	Module 3	Module 4
Geometry	Patterning	Numeracy	Computation
• Shape Recognition • Shape Matching • Shape Properties • Composing and Decomposing Shapes • Disembedding Shapes	• Imitating Patterns • Recognizing Patterns • Reproducing Patterns • Extending Patterns • Creating Patterns • Fixing Patterns • Translating Patterns	• Recognizing Quantities • Counting and Cardinality • Subitizing • Comparing and Ordering Numbers • Composing and Decomposing Numbers • Recognizing and Writing Numerals • Changes in Sets • Equipartitioning (Sharing Fairly)	• Addition • Subtraction • Part-Part-Whole • Fractions

EMAS development over time

Pre-kindergarten and Kindergarten Expansion

When VKRP began in 2014, the original EMAS developed by Ginsburg and colleagues (2010) was used to assess students' mathematics skills at the start of kindergarten. The gradual statewide rollout of VKRP allowed teachers and administrators to provide feedback about the EMAS. The most common feedback from teachers suggested that the EMAS should be 1) expanded from pre-kindergarten through the end of kindergarten; 2) more sensitive to the performance of students with average or advanced skills to better understand the needs of students across the developmental continuum; and 3) better aligned to the Virginia Standards of Learning (SOLs).

From 2017 on, the VKRP team set out to make these revisions to the EMAS. To expand the assessment, the team:

- Developed or adapted additional items capturing Numeracy, Computation, Geometry, and Patterning;
- Consulted with early childhood mathematics experts and colleagues at the Virginia Department of Education (VDOE);
- Cross-walked each item with Clements' and Sarama's (2021) learning trajectories, the Virginia Early Learning and Development Standards (2021), as well as the 2016 and 2023 Kindergarten Math Standards of Learning, and recent early childhood mathematics research. Because the ELDS and SOLs focus on end-of-year goals, VKRP drew on the other sources to make sure we measure precursor skills along the way to mastery of the standards.
- Sought input from a nationally recognized experts on educational measurement;
- Pilot-tested each new item;

- Used state-of-the-art statistical methods to analyze each item, examining, for example, how students of different background demographics responded to each item;
- Triangulated a) expert opinion, b) content crosswalks, and c) statistical result to create a vertically aligned EMAS that spans from the fall of pre-kindergarten (3-year-old) to the end of kindergarten.

Psychometric Properties of the Revised VKRP EMAS

Item-response theory (IRT) is an advanced statistical approach that allows us to determine whether the range of difficulty of the items is appropriate for the students being assessed and provides an additional estimate of the measure's reliability.

After data were gathered on the new items, researchers combined this new data with data from students assessed with the original EMAS. Based on a careful analysis of each item, the research team constructed nine EMAS forms: a fall, mid-year, and spring version of Pre-k 3, Pre-k 4, and Kindergarten EMAS assessments. Items were selected to represent a range of skills across the four subdomains and to target an appropriate average level of difficulty for each age level. The research team also deliberately selected some easier and some more challenging items for each level so that teachers can gauge which students need extra support and which may need additional challenge.

Lastly, the research team converted the new EMAS scores into growth scores so that teachers and schools/programs can track students' mathematics growth over time. Different versions of EMAS tests have different items on them and different numbers of subdomain items, so raw scores cannot be compared directly across age levels. However, the scaled growth score is calculated based on the results of the IRT analyses and puts the results from each of the forms on a standard scale, enabling comparison of growth scores within and between students.

In summary, the VKRP research team undertook a careful process to develop a new and expanded set of EMAS assessments to track growth in pre-kindergarten and kindergarten students over time. These new assessments show robust measurement properties, are quick to administer, and are engaging for students.

EMAS Form Breakdown

Table 1

Summary Statistics of the Pre-k 3, Pre-k 4 and K EMAS Forms

Grade and Window	Number of Items	Internal Reliability (Cronbach's Alpha)
Fall Pre-k 3	26	0.912
Spring Pre-k 3	30	0.918
Fall Pre-k 4	33	0.930
Spring Pre-k 4	35	0.922

Grade and Window	Number of Items	Internal Reliability (Cronbach's Alpha)
Fall K	35	0.919
Spring K	34	0.909

Frequently Asked EMAS Questions

Why are there items with a range of difficulty on the EMAS?

Children enter each grade level with a wide range of skills. The EMAS includes easy, medium, and difficult items to observe what children know and can do mathematically. If the items are too easy, we miss identifying children who need more challenge. If the items are too difficult, we miss children who need more support.

Imagine a child who starts kindergarten and can count to three, but she can't yet count to 10. She can point to a square and a triangle, but she can't consistently name any shapes without help. If the assessment is too hard, she may get all or most of the items incorrect. Even after the assessment, her teacher will have very little information about what she *can* do or how to plan instruction to meet her needs.

Imagine another child who can count to 100 and name advanced shapes like "hexagon" and "oval". If the assessment is too easy, she will get nearly all the items correct. Again, her teacher will have very little information about what she can do, because none of the questions challenged her.

The EMAS includes a broad range of items so that teachers get the information they need to understand what each student knows and can do.

How can you measure growth over time with the EMAS?

In general, longer tests tend to be more informative and accurate because they include many easy, medium, and difficult items that cover a range of skill levels; and when you give children the same assessment twice a year, you can easily track growth. However, a single test that covered easy through difficult items for children in the fall *and* spring would be very long! And that type of assessment *still* wouldn't be able to track growth over multiple years.

To solve these challenges, the VKRP team created a series of assessment versions that each include some shared items and some unique items. The assessments get a little more difficult with each version. For example, the spring pre-kindergarten EMAS assessment is a little more challenging than the fall pre-kindergarten EMAS assessment, because children grow in their mathematics knowledge and skills over the year.

Using data from the overlapping items, scores on each assessment version are converted mathematically to be on the same numerical scale ("scaled scores"). This enables teachers to compare scores from the fall to scores in the spring to see how much each child's scores grew. Administrators can also compare performance over time; for example, looking at how children grow in their mathematics skills from pre-kindergarten through kindergarten.

Common, widely used assessments, like the NWEA MAP measure, use similar approaches to keep assessments as short as possible while measuring growth over time.

Why aren't the kindergarten EMAS benchmarks fully aligned to the kindergarten SOLs?

The EMAS was not designed to be an SOL test, so it includes a broader range of items than just those included in the SOLs. Some of the items are included because they are foundational for mathematical thinking. Research shows that these skills are important for children's mathematical learning far into the future. For example, subitizing (quickly recognizing small quantities without counting) is not on the 2023 Kindergarten SOLs. However, subitizing is a critical skill that underlies mathematical thinking, though, and it is identified as a precursor to numeracy within the "Understanding the Standards" document for kindergarten mathematics. VKRP includes this skill on the assessments because difficulty with subitizing could help identify children who need extra support in mathematics.

The VKRP kindergarten benchmarks were developed using a process that includes triangulating between the following pieces of information:

a. EMAS data from the Commonwealth of Virginia;
b. the Mathematics Learning Trajectories (Sarama & Clements, 2009; Clements & Sarama, 2021);
c. the Virginia Mathematics Standards of Learning (2016; 2023) and Early Learning and Development Standards (2021); and
d. a review of the most recent early childhood mathematics research.

Therefore, there is not an exact 1:1 correspondence between the SOLs/ELDS and the EMAS benchmark.

Appendix A: Kindergarten EMAS Items and ELDS/SOL/Trajectory Alignment

Sample Items				
Domain	**Skill**	**Item**	**ELDS[3]/SOL[4]**	**Mathematics Learning Trajectories[2]**
Geometry	Shape Recognition	Recognize and name a rectangle	Correctly names squares, rectangles, and triangles regardless of size or orientation (CD3.3t) Identify and name concrete and pictorial representations of circles, triangles, squares, and rectangles regardless of their orientation in space (K.MG.2.a)	*Shape Recognizer – All Rectangles:* Recognizes and names polygons with four sides and four right angles as "rectangle", regardless of size, shape, and orientation.
	Shape Properties	Recognize shapes with 4 equal sides (square)	Describe triangles, squares, and rectangles to include the number of sides and number of vertices (K.MG.2.b)	*Side Recognize:* Recognizes sides and angles (referred to as corners) as geometric properties.
	Composing Shapes	Composing a new shape out of smaller shapes (square)	Uses smaller shapes to compose larger and different shapes (e.g., two triangles make one square) (CD3.3s)	*Shape Composer:* Composes new shapes by flipping and rotating smaller shapes, attending to their sides and angles.
Patterning	Recognizing Patterns	Recognize ABAB pattern ("Here are two groups of pictures. Which one is a repeating pattern?")	Shows understanding of simple patterns by recognizing and extending simple, repeating, "ABAB" patterns (CD3.4j)	*Pattern Recognizer:* Recognizes a simple sequential pattern, usually ABAB.

[3] Virginia Board of Education. (2021). Virginia's Early Learning & Development Standards. https://www.doe.virginia.gov/home/showpublisheddocument/421/637890605072570000
[4] Virginia Board of Education. (2023). Mathematics Standards of Learning for Virginia Public Schools. https://www.doe.virginia.gov/home/showpublisheddocument/48908/638550125418770000

	Reproducing Patterns	Reproduce ABAB pattern	Extend a repeating pattern by adding at least two complete repetitions of the core to the pattern (K.PFA.1.b)	*Patterner:* Recognizes, describes, and builds repeating patterns. This includes AB and ABB sequences, as well as sequences with three core units, such as ABC and AABC.
	Extending Patterns	Extend ABBABB pattern	Create and describe a repeating pattern using objects, colors, sounds, movements, or pictures (K.PFA.1.c)	Extend more complex patterns, such as ABBABB by adding on several ABB units to the end of the pattern
Numeracy	Counting and Cardinality	One-to-one counting (6 chips)	Shows accuracy in demonstrating one-to-one correspondence for up to 10 objects (CD3.1s) Use one-to-one correspondence to determine how many are in a given set containing 30 or fewer concrete objects (e.g., cubes, pennies, balls), and describe the last number named as the total number of objects counted (K.NS.1.a)	*Counter (10):* Counts sets of objects to ten with an understanding of the cardinality principle.
	Subitizing	Subitizing – 6 dots ("How many dots did you see?")	Instantly recognizes a collection of up to 10 objects (i.e., subitizes) (CD3.1w)	Perceptual/Conceptual Subitizer: Identifies all arrangements to 5, then 6 when shown only briefly, by seeing the parts and quickly knowing the whole.
	Describing Sets	Changes in sets (more)	Compares sets of objects that range in size from 1-5, as having "more" or "fewer" (CD3.1p)	*Counting Comparer (Same Size):* Accurately compares groups of 1-6 through counting when

			Compares sets of objects that range in size from 1-10, as having “more”, “fewer” or “same” (CD3.1z) Compare two sets containing up to 30 concrete objects or pictorial models, using the terms more, fewer, or the same as (equal to) (K.NS.2.f)	objects are roughly the same size.
	Recognizing and Writing Numerals	Write numerals to represent a quantity - 5	Read, write, and identify the numerals 0 through 30 (K.NS.2.a)	*Counter (10):* May be able to read and write numerals to represent 1-10.
	Composing Numbers	Show ways to make 7 (“Can you show me a different way I can make 7 by putting some chips in one pile and some chips in another pile?”)	Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 5 in multiple ways (K.CE.1.a) Use objects, drawings, words, or numbers to compose and decompose numbers less than or equal to 10 in multiple ways (K.CE.1.d)	*Composing to 7:* Knows number combinations to totals of 7, quickly naming parts of any whole, or the whole given parts.
Computation	Addition	Solve part-part-whole, result unknown (“Bear has 5 yellow balloons. His mother gives him 4 red balloons. How many balloons does Bear have now? 5 plus 4 is how many altogether?”)	Solves addition (joining) problems using manipulatives (e.g., fingers, objects, tally marks) (CD3.2c) Model and solve single-step contextual problems (join, separate, and part-part-whole) using 10 or fewer concrete objects (K.CE.1.f)	*Find Result (+/-):* Finds results for (a) Join, Result Unknown (b) Separate, Result Unknown, and (c) Part-Part-Whole, Whole Unknown problems. This is accomplished using direct modeling and counting all strategies.

	Subtraction	Subtraction with ten frame ("Here are five cookies. If you take two away, how many cookies would you have left?")	Solves subtraction (separating) problems using manipulatives (e.g., fingers, objects, tally marks) (CD3.2d) Model and solve single-step contextual problems (join, separate, and part-part-whole) using 10 or fewer concrete objects (K.CE.1.f)	*Small Number (+/-):* Can separate and solve result unknown problems for totals up to 5, by using the count all strategy.

Appendix B: Pre-k 3 EMAS Items and ELDS/Trajectory Alignment

Sample Items				
Domain	**Skill**	**Item**	**ELDS[3]**	**Trajectory[2]**
Geometry	Shape Recognition	Recognize and name a square	Recognizes and names a typical circle, square, and sometimes triangle (CD3.3m)	*Shape Recognizer – Typical:* Recognizes and names typical circle, square, and sometimes triangle. May physically rotate shapes presented in atypical orientations to visually match them to a prototype.
	Shape Matching	Match two triangles	Matches some shapes that are different sizes or orientations with adult support (CD3.3n)	*Shape Matcher:* Matches familiar shapes that have the same size and orientation.
Patterning	Recognizing Patterns	Recognize ABAB pattern ("Here are two groups of pictures. Which one is a repeating pattern?")	Shows understanding of simple patterns by recognizing and extending simple, repeating, "ABAB" patterns (CD3.4j)	*Pattern Recognizer:* Recognizes a simple, sequential AB pattern.
	Reproducing Patterns	Reproduce ABAB pattern		*Patterner AB:* Recognizes, describes, and builds repeating patterns, including AB but also patterns with core units such as AAB, ABC, and AABC.
	Extending Patterns	Extend ABAB pattern		
Numeracy	Counting and Cardinality	Count one-to-one (4 chips)	Begins to explore one-to-one correspondence (e.g., setting places at a table), counting up to 10 (CD3.1n) Shows accuracy in demonstrating one-to-one	*Corresponder:* Keeps one-to-one correspondence between counting words and objects (one number word for each object), at least for small groups of objects laid in a line.

			correspondence for up to 10 objects (CD3.1s)	May answer a "How many?" question by recounting the objects or violate 1-1 or word order to make the last number word be the desired or predicted word.
	Subitizing	Subitizing – 4 dots ("How many dots did you see?")	Instantly recognizes a collection of up to 4 objects (i.e., subitizes) (CD3.1m)	*Perceptual Subitizer to 4:* Instantly recognizes collections up to 4 briefly shown and verbally names the number of items.
	Comparing and Ordering Numbers	Changes in sets (more)	Compares quantities of items and indicates "same" or "more" (CD3.1f)	*Early Comparer of Similar Items:* Compares collections of 1-4 items verbally and nonverbally ("just by looking"). Items must be the same. May compare small collections using number words.
Computation	Addition	Adding with objects ("Bear has 3 cookies. His mom gives him 1 more. How many cookies does Bear have altogether?")	With adult help, adds (joins) two small sets of objects (e.g., "I have two books and you have two books. How many books do we have all together?") (CD3.2a)	*Small Number (+/-):* Finds the sum for joining and result unknown problems for totals up to 5, by using the count all strategy. *Small Number (+/-):* Can separate and solve result unknown problems for totals up to 5, by using the count all strategy.
	Subtraction	Subtraction with ten frame ("Here are three cookies. If you take one away, how many cookies would you have left?")	Solves subtraction (separating) problems using manipulatives (e.g., fingers, objects, tally marks) (CD3.2d)	

Appendix C: Pre-k 4 EMAS Items and ELDS/Trajectory Alignment

Sample Items				
Domain	**Skill**	**Item**	**ELDS[3]**	**Trajectory[2]**
Geometry	Shape Recognition	Recognize and name a rectangle	Correctly names squares, rectangles, and triangles regardless of size or orientation (CD3.3t) Recognizes and names a typical circle, square, and sometimes triangle (CD3.3m)	*Shape Recognizer – Typical:* Recognizes and names typical circle, square, and sometimes triangle. May physically rotate shapes presented in atypical orientations to visually match them to a prototype.
	Shape Properties	Identify the number of angles of a triangle	Describes attributes of two- and three-dimensional shapes (e.g., “A square has four corners/angles”, “a triangle has three straight sides”) (CD3.3u)	*Corner (Angle) Recognizer:* Recognizes angles as a separate geometric object.
Patterning	Recognizing Patterns	Recognize ABAB pattern (“Here are two groups of pictures. Which one is a repeating pattern?”)	Shows understanding of simple patterns by recognizing and extending simple, repeating, “ABAB” patterns (e.g., of movements such as “tap head, tap knees, tap head...”; or of objects such as “red car, yellow car, red car...”) (CD3.4j)	*Pattern Recognizer:* Recognizes a simple, sequential AB pattern.
	Reproducing Patterns	Reproduce ABBABB pattern		*Patterner AB:* Recognizes, describes, and builds repeating patterns, including AB but also patterns with core units such as AAB, ABC, and AABC.
	Extending Patterns	Extend ABBABB pattern		

Numeracy	Counting and Cardinality	Count one-to-one (6 chips)	Begins to explore one-to-one correspondence (e.g., setting places at a table), counting up to 10 (CD3.1n) Shows accuracy in demonstrating one-to-one correspondence for up to 10 objects (CD3.1s)	*Counter (10):* Counts sets of objects to ten with an understanding of the cardinality principle.
	Numerals	Identify the number 5	Names some numbers when they appear in print (CD3.1j)	*Counter (10):* Counts arrangements of objects to 10 with understanding of the cardinal principle. May be able to read and write numerals to represent 1-10.
	Subitizing	Subitize (4 dots)	Instantly recognizes a collection of up to 4 objects (i.e., subitizes) (CD3.1m)	*Perceptual Subitizer to 4:* Identifies all arrangements to 4 when shown only briefly, by seeing the parts and quickly knowing the whole.
Computation	Describing Changes in Sets	Determine changes in sets (more)	Compares sets of objects that range in size from 1-5, as having "more" or "fewer" (CD3.1p)	*Early Comparer of Similar Items:* Compares collections of 1-4 items verbally and nonverbally ("just by looking"). Items must be the same. May compare small collections using number words.

Computation (continued)	Addition	Adding with objects ("Bear has 3 cookies. His mom gives him 1 more. How many cookies does Bear have altogether?")	Solves addition (joining) problems using manipulatives (e.g., fingers, objects, tally marks) (CD3.2c)	*Small Number (+/-):* Finds the sum for joining and result unknown problems for totals up to 5, by using the count all strategy.
	Subtraction	Subtraction with ten frame ("Here are three cookies. If you take one away, how many cookies would you have left?")	Solves subtraction (separating) problems using manipulatives (e.g., fingers, objects, tally marks) (CD3.2d)	*Small Number (+/-):* Can separate and solve result unknown problems for totals up to 5, by using the count all strategy.

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